

UNDERSTANDING Interest Rates IN CASH VALUE LIFE INSURANCE



by
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Cash value life insurance policies can be very complicated, making it difficult for the newcomer to evaluate claims made about these mysterious creatures. One of the chief ambiguities concerns the distinction between the “guaranteed interest rate” and/or “credited interest rate” on a cash value policy, versus the very familiar concept of “internal rate of return” on more traditional financial products.

I know from personal experience that I was briefly indignant when a representative from my insurance company told me on the phone that I had a certain “guaranteed interest rate” on the cash value of my whole life policy, because I had earlier worked up an Excel spreadsheet and seen that my policy illustrations showed no such return, not even by the 40th year of the policy. The representative tried to explain to me what the “guaranteed interest rate” really meant—hint: it’s *not* the internal rate of return on the gross premium payments—but I

nonetheless left that phone call with a bad taste in my mouth. I thought it was a very misdealing term to be throwing around, since it didn’t mean what the average person would *think* that it meant.

Now that I have studied more of the actuarial science behind permanent life insurance policies, I understand why the representative thought he was being quite helpful and truthful in what he said. Even so, it’s important for owners and especially agents to understand at least the basic mechanics of what makes these policies tick. The present article will be somewhat academic in nature, but I hope that going through the process step-by-step will shed light on this potentially confusing topic.

Permanent Life Insurance: Where the Simple Becomes Complex

In principle, a cash value policy such as an ordinary whole life policy is a simple thing: The policyowner agrees to pay a stream of premium payments to

the insurer so long as he is still alive, while the insurer agrees to pay a stated death benefit upon death or upon the attainment of a certain age (such as 100 or 121).

However, in practice even a plain vanilla whole life policy becomes difficult to evaluate quantitatively, because it involves two moving parts, as it were: (1) discounting future cash flows and (2) taking into account the uncertainty of death, which will greatly influence the composition of those future cash flows.

In order to shed light on the terminology and behavior of permanent life insurance, in this essay I’ll start from an easy case and then build upwards. Our first stop is the analysis of a simple bond.

Baby Step 1: A Simple Bond With Various Discount Rates

First let’s focus purely on the time factor. Suppose a financial institution tells a man, who happens to be 35 years old, that it will pay him \$1,000 in exactly

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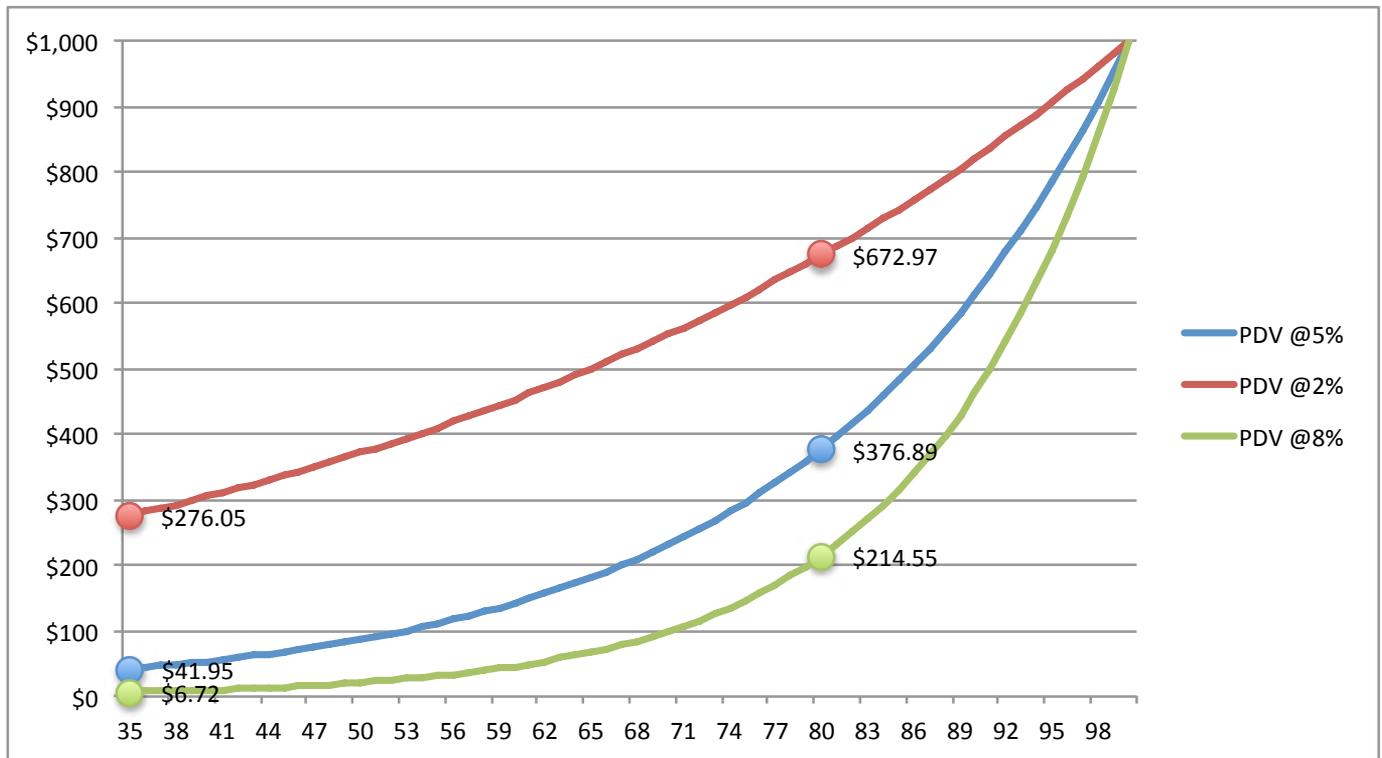


FIGURE 1. Present Discounted Value (PDV) of \$1000 Bond at Different Discount Rates

sixty-five years, when the man will happen to be 100 years old. Now the question is, how much should the man value that promise *right* now? Another way of putting it is to ask, if the man can sell this IOU from the company, how much would he be able to fetch for it in the marketplace? Let's take risk out of the analysis

entirely, and assume that no one has any doubt whatsoever that the company will be around in sixty-five years, and that it will indeed honor its promise to pay \$1,000 at that time.

Clearly the IOU—or what we will call a bond from now on—isn't currently worth

the full \$1,000, because a dollar *today* is more valuable than a dollar that will only be delivered decades in the future. That means we have to *discount* that future \$1,000 payment.

In order to calculate a total discount for the entire period, it

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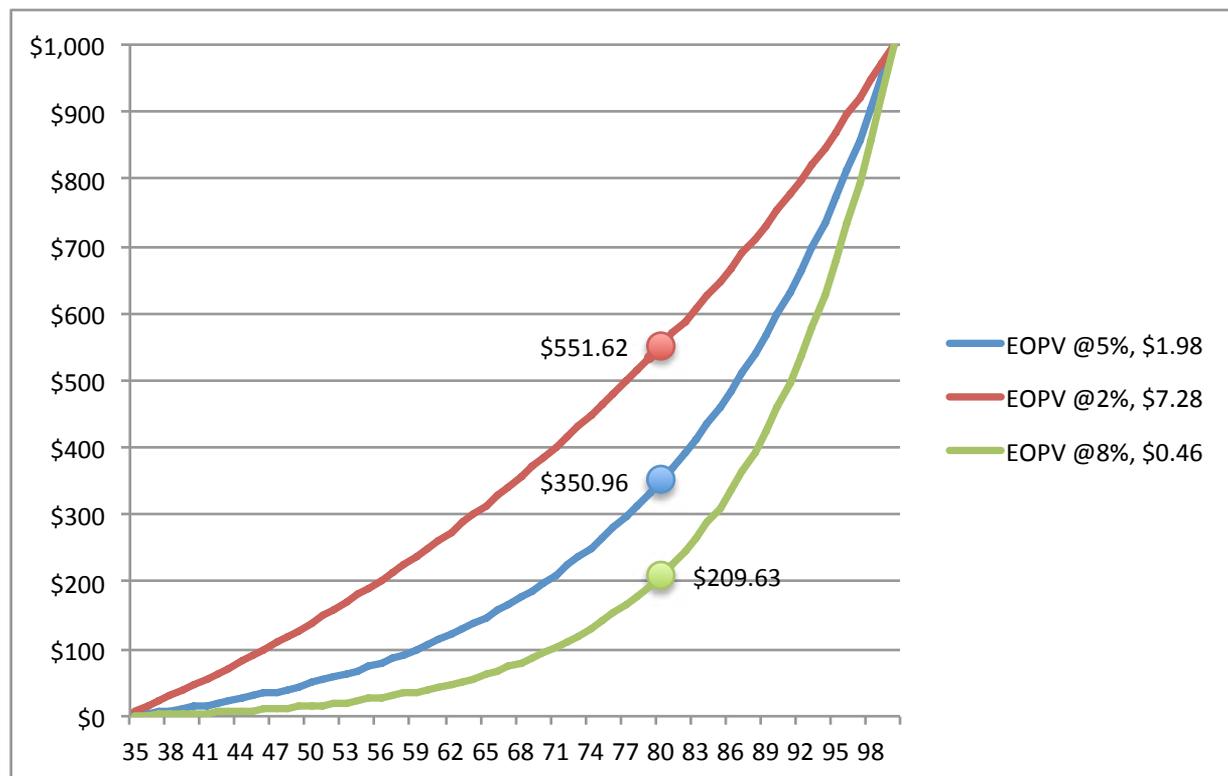


FIGURE 2. End-of-Period Value of \$1000 Bond With Various Discount Rates and Implied Level Annual Payments

is standard practice to assume the man uses an average annualized discount rate. Figure 1 shows the present discounted value (PDV) of the bond, at various points in the man's life, at three different discount rates.

Figure 1 shows three different trajectories for the "present discounted value" of the \$1,000 bond, as the owner of the bond ages. At a 2% discount rate, that future \$1,000 payment is valued more highly in earlier time periods—it is discounted less than in the other scenarios. That's why the red line is consistently higher over the man's life, until finally in the 100th year the other lines finally catch up to it.

Notice that *if we fix the*

ultimate payout, then there is tradeoff between the height of the PDV at any time, and the rate of its growth. In other words, the red line is always higher than the other two lines, but the bond's value in that trajectory grows the most slowly (at only 2% per year). In contrast, the green line is consistently below the other two lines, yet the bond's market value grows very quickly here (8% per year).

Even with this simple bond example, we can illustrate a distinction that comes up in the analysis of life insurance: calculating the present value of an asset using either a *prospective* or a *retrospective* approach. In our example above, consider the market value of the bond

at age 80. At a discount rate of 2%, Figure 1 tells us that the value is \$672.97. There are two (equivalent) ways of arriving at this figure. In the prospective approach, we look at *future events* and use them to determine the present value. In this simple case, the only cash flow that will occur is a payment of \$1,000 to the man, which will happen (from his perspective at age 80) in twenty years. If we divide that \$1,000 payment by $((1.02)^{20})$, we end up with \$672.97. To repeat, this is the *prospective* approach.

On the other hand, we could reach the same figure by the retrospective approach. The original market value of the bond—what the man would have had to pay for it at auction—was

\$276.05. That original investment then grew at a 2% compounded annual rate for forty-five years, so its present value is \$276.05 times $((1.02)^{45})$, or \$672.97.

Thus we see that the prospective and retrospective approaches yield the same current market value, at least if we assume nothing relevant changes during the man's lifetime regarding the discount rate or the cash flows associated with the bond.¹

Baby Step 2: Level Contributions for a Certain Payout

Now let's introduce another layer of complexity, inching us closer to our ultimate goal of a whole life insurance policy. In this baby step, the financial institution is still promising to pay the man \$1,000 when he reaches age 100. In this scenario, however, he is obligated to make a level stream of annual payments to the company from age 35 onward, in order to remain eligible for the \$1,000 payout. We'll further assume that initially the bond has zero value.

So now the question is, what does the level payment need to be, at each of the hypothetical discount rates, in order to make the bond start at \$0 at age 35,² and end up at \$1,000 by age 100?

By playing with an Excel spreadsheet, one can zoom in to find that the level payments are \$7.28, \$1.98, and 46 cents if we use discount rates of 2%, 5%, and 8%, respectively. Figure 2 shows the trajectories of their market values in this new setting.

Here too we can use either the prospective or retrospective method to calculate the market value at any particular age, though the calculations are trickier. The retrospective method is quite intuitive, since the asset in this case behaves just like a savings account with a conventional bank, which is growing at interest while the man continually pumps in more saving each year. For example, using the 5% discount rate, pumping \$1.98 in at the beginning of each year to add to the previous year's end-of-period market value, and then letting

the whole sum grow 5% during the current year, will lead to an end-of-period value of \$350.96 at age 80.

We can get the same result by forgetting the past, and just focusing on the future (i.e. by using the prospective approach). If the man at the end of his 80th year evaluates the future cash flows, he sees that he will receive \$1,000 from the financial institution in 20 years (I assume the payment comes at the end of the year). Thus the benefit is worth \$376.89—just as we reckoned in the earlier section, when calculating the PDV of a simple bond.

However, in our new scenario, this number would be overstating the value of the asset. In order to get his hands on that \$1,000 payment when he is 100 years of age, the man must continue to make his level \$1.98 contributions for the next twenty years, as well. From his vantage point at the end of his 80th year, the present value of *that* stream of contributions—discounting

NOW, PUT YOURSELF IN THE POSITION OF THE financial institution—which at this point we might as well start calling “the insurance company.” If the man wants to pay a level premium, what do you charge him to make sure you cover yourself?

at 5%—is \$25.93.³ Thus, the *net* value of the asset is only \$376.89 - \$25.93 = \$350.96. As before, the retrospective and prospective approaches yield the same answer for the current value of the asset.

Baby Step 3: Introducing the Risk of Death, But With Insurer Overcharging

Now we're ready to drop the unrealistic assumption that the man would necessarily live to age 100. To make our lives easier when doing the math, assume (quite unrealistically) that every year, there is a 1% probability the man will die. Thus his mortality risk stays exactly the same, throughout his whole life. Further assume that the asset now promises to pay the man \$1,000 *either* upon death, *or* at the end of age 100, if he still happens to be alive at that point. As before, the man has to make level contributions to the financial institution, in order to remain eligible for these \$1,000 payment possibilities.

At this point, the analysis is going to get more complicated so let's drop the three different discount rates, and just work with a 5% rate to keep things simple on that score. Now, put yourself in the position of the financial institution—which at this point we might as well start calling “the insurance company.” If the man wants to pay a level premium, what do you charge him to make sure you cover yourself?

We already know from the previous section that if the man

would be *certain* to live to age 100, then the break-even premium (using a 5% discount rate) is \$1.98 per year. Essentially, the insurance company takes those \$1.98 premiums and invests them in the marketplace earning 5% per year, and accumulates a fund that is exactly equal to \$1,000 at the end of the man's 100th year.

Yet if you the insurer only charged the man \$1.98 in the new scenario, you could lose money on him. Every period, there is a 1% chance that he'll die. Such an outcome is a double whammy for you, the insurer. For example, if the man dies at age 75, not only do you have to pay the \$1,000 twenty-five years earlier—which therefore represents a greater burden to you, since earlier dollars are worth more than later dollars—but you also miss out on twenty-five years' worth of \$1.98 premium payments. How should you, the insurer, deal with this tricky situation?

One way (which gives too high an answer, as we'll see in a minute) is to have the insurer slap on a pure term insurance premium, in addition to the underlying \$1.98 that is necessary to fund the payment at age 100. Every year, there is a 1% probability that the man will die, requiring \$1,000 at that time. Thus, the actuarially fair pure term insurance premium each year is \$10.

Therefore, you the insurer would certainly be covering yourself (disregarding overhead and other business expenses)

on the pure financing of the contractual obligations, by charging the man a total premium of \$1.98 + \$10.00 = \$11.98 each year. This way, if he dies you're covered by the \$10 term payments each year, and even if he survives to 100 then you've been collecting \$1.98 and investing it on his behalf for sixty-five years. No matter what happens to the man, you will be covered and can pay him. (We are assuming of course that you have a large pool of similar customers, so that by charging each of them \$10 per year in pure term premiums, you will have the cash flow to make the death benefit claims to the 1% of the pool who happen to die that year.)

But wait a second. The \$11.98 premium is actually too high. You the insurer really only “break even” (again, disregarding other business expenses) on this arrangement if the man lives to 100. If he dies at any earlier point, you the insurer have strictly benefited from the deal, because *you get to keep the accumulating fund* that had been earmarked for his possible attainment of age 100.

This fund has the same market value as depicted in the blue line in Figure 2 above. (Remember, each period \$10 of the man's gross premium is used to pay the death benefits of other people in his pool, who happened to die that year. That's what the term premium is doing, from an actuarial accounting standpoint; that money is already spoken for.)

Age	Cash Value Before Mortality Charge (bop)	Net Amount at Risk (bop)	Premium Absorbed By Mortality Expense	Premium "Going Into Cash Value"	Cash Value After Mortality Charge and 5% Credited Interest (eop)
35	\$0.00	\$1,000.00	\$10.00	\$1.22	\$1.28
36	\$1.28	\$998.72	\$9.99	\$1.23	\$2.64
37	\$2.64	\$997.36	\$9.97	\$1.25	\$4.08
38	\$4.08	\$995.92	\$9.96	\$1.26	\$5.60
39	\$5.60	\$994.40	\$9.94	\$1.27	\$7.22
40	\$7.22	\$992.78	\$9.93	\$1.29	\$8.94
...
65	\$102.07	\$897.93	\$8.98	\$2.24	\$109.53
66	\$109.53	\$890.47	\$8.90	\$2.31	\$117.44
67	\$117.44	\$882.56	\$8.83	\$2.39	\$125.82
68	\$125.82	\$874.18	\$8.74	\$2.48	\$134.71
69	\$134.71	\$865.29	\$8.65	\$2.57	\$144.14
70	\$144.14	\$855.86	\$8.56	\$2.66	\$154.14
...
95	\$696.69	\$303.31	\$3.03	\$8.19	\$740.12
96	\$740.12	\$259.88	\$2.60	\$8.62	\$786.17
97	\$786.17	\$213.83	\$2.14	\$9.08	\$835.02
98	\$835.02	\$164.98	\$1.65	\$9.57	\$886.81
99	\$886.81	\$113.19	\$1.13	\$10.09	\$941.75
100	\$941.75	\$58.25	\$0.58	\$10.64	\$1,000.00

TABLE 1. Values at Various Ages Using Net Amount at Risk (NAR) Approach (premium=\$11.22, bop=beginning of period, eop=end of period)

For example, suppose our man dies at age 80. You the insurer can pay his beneficiary the \$1,000 out of the \$10 term premiums collected from everyone in the pool of customers that year, leaving the \$350.96 (which had been accumulating from the \$1.98 portion of the premiums since age 35) free and clear. The longer the man lives—and each year, he has a 99% probability of continuing on for another—the

larger the fund grows.

So if \$1.98 is too low a premium, and \$11.98 is too high, how do you the insurer figure out the exact actuarially fair amount to charge the man, for what is now an ordinary whole life insurance policy that completes at age 100?

Baby Step 4: Introducing “Net Amount at Risk” (NAR) Approach

Actuaries have a very elegant solution to this pricing problem. The mistake we made in the previous section was to charge the break-even term premium for *the full \$1,000* every year. Instead, all you as the insurer need to do is charge the term premium on *the current difference between the death*

benefit and the accumulating fund. In other words, in a given year you should charge the man (a) the \$1.98 premium to continue growing the fund that endows at age 100, plus (b) the term premium for a one-year policy that has a death benefit equal to the “net amount at risk” (NAR), which is the difference between \$1,000 and the fund’s present market value.

Table 1 shows these calculations for the beginning and ending years of the man’s potential life, again assuming a 5% discount rate.

Let me offer some commentary to be sure you understand how to read Table 1. At the beginning of the policy at age 35, the man makes his level premium payment of \$11.22. At this point, there is no fund to offset a death claim, so the entire \$1,000 death benefit is “at risk.” Consequently, because there is a 1% chance of death this year, the insurer must devote \$10 of the premium payment just to pure

term insurance. This leaves only \$1.22 available to invest on behalf of this particular client. Since the premium payment is collected upfront, and since cash values grow at 5% annually, by the end of age 35 the \$1.22 has grown into \$1.28. If the man for some reason decided to surrender the policy at this point, the insurer could hand him \$1.28 and break even on the whole deal—again, unrealistically assuming away all of the other real-world expenses involved with issuing insurance policies.

At the beginning of age 36, the man again pays his level premium of \$11.22. This time, however, the full \$1,000 isn’t at risk—the insurer now has a dinky little fund of \$1.28. Consequently, if the man happened to die this year, the insurer would only need a term policy with a face amount of $\$1,000 - \$1.28 = \$998.72$ to pay the death claim. The actuarially fair premium for this term policy is $(1\% \times \$998.72) = \9.99 with rounding, which is one penny lower than the full \$10 that was

needed at age 35. The extra cent goes into the man’s accumulating fund, which grows at 5% again.

In case it’s not clear, I should explain that I set up an Excel spreadsheet with the above framework, and then simply experimented with the level premium payments until I got the age 100 end-of-period cash value to equal \$1,000.00. That’s where the \$11.22 level premium came from.

At Last: Guaranteed Interest Rate vs. Internal Rate of Return

We can now, at long last, easily see the distinction between the interest rate credited to the cash value of a permanent insurance policy, versus the calculated “internal rate of return” on the *gross* premiums associated with the policy.

By construction, the cash values in our hypothetical ordinary whole life policy in Table 1, grew at 5% throughout

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the life of the policyowner. If the man had called the insurer and asked, “How much am I earning on my policy, considered as an investment?” the representative could quite honestly tell him, “We are crediting your assets with a 5% annual growth.”

However, if the man completely disregarded the *insurance* aspect of his policy, and looked at its surrender cash values purely as a mutual fund, then he would be appalled at its performance. With the particular numbers I chose for our example, the calculated internal rate of return (IRR) on this policy is only 0.86% by age 100. In other words, if the man started at age 35 and put \$11.22 each year into a savings account, such that his balance were \$1,000 by the end of age 100, then the bank would only have to pay him a compounded annual rate of interest of 0.86%.

Since the market rate of interest in our example is 5%, the man would presumably be

outraged by this result, *if he totally disregarded the insurance element*. But it would be completely inappropriate to treat his whole life policy as a mere mutual fund, since it is so much more than that. Yes, by making \$11.22 annual contributions, the man is assured of a \$1,000 payout at age 100—just as he would be assured, doing the same activity, with a bank paying 0.86% on its saving accounts. Yet with the insurance policy, if the man dies just after turning 36, he *also* gets the full \$1,000. In contrast, he will only have \$23 with the bank.

I want to stress that in the real world, a typical whole life policy has a much higher internal rate of return (IRR) than the measly 0.86% in this hypothetical example. That calculation comes from the nice round numbers I picked to make the math easy in Table 1. The point of this example is to teach you the distinction between the “guaranteed interest rate” versus the internal rate of return on

the gross premiums. For realistic comparisons of a mutual fund versus a whole life policy, you should consult an actual policy illustration generated by an agent who knows how to properly design a whole life policy.

Conclusion

Although this article was long and heavy on the numbers, I hope it helped some readers to finally grasp exactly what is going on “under the hood” with cash value life insurance policies. Obviously my explanation left out many important real-world considerations, such as expense loading, changing mortality rates, and “adverse selection” based on changing insurability status. Even so, the above progression of scenarios should shed light on how actuaries use discount/interest rates to calculate the current cash value of a policy.



Notes

1. If things do change, once the man begins moving down the trajectory, then economists would strictly prefer using the prospective approach, because “bygones are bygones” and all that matters right now when evaluating an asset, is what the owner thinks it will do for him going forward. But so long as nothing important changes along the way, then these correct on-the-spot calculations have already been anticipated beforehand, and so the two approaches give the same answer.

2. For the purist who might actually try to replicate my results, I should mention that these assets are only worth \$0 at the beginning of age 35. I am assuming that the man puts in his level premium payment in the beginning of the year, and so by the end of age 35, the premium payments have grown at the respective interest rates, giving market values of \$7.42, \$2.08, and 50 cents for the three rates.

3. The present discounted value stream of contributions looks like this: $\$1.98 + \$1.89 + \$1.80 + \dots + \$0.82 + \$0.78 = \25.93 . Note that the age-81 contribution of \$1.98 is not discounted by 5%, because the man reckoning at the end of age 80 is just about to make this particular payment.